

when  $\nu = 2$ . Also, the interval  $0 \leq \xi \leq 0.1$  has now been divided into 10 sub-intervals. However, as in the earlier version of Table 2, the critical values corresponding to  $\xi$  between 0.01 and 0.2 have been progressively omitted as  $\nu$  increases to 14, because of corresponding loss of precision in the calculations.

Expansion of these tables was necessary in order to obtain the author's new tables of standard confidence limits, which will be described in a subsequent review.

J. W. W.

1. CHARLES E. LAND, *Tables of Critical Values for Testing Hypotheses about Linear Functions of the Normal Mean and Variance*, Department of Statistics, Oregon State University, Corvallis, Oregon, ms. deposited in the UMT file. (See *Math. Comp.*, v. 25, 1971, p. 941, RMT 44.)

43 [9].—JACK ALANEN, *Empirical Study of Aliquot Series*, Report MR 133, Stichting Math. Centrum, Amsterdam, x + 121 pp., July 1972.

This is essentially a reprint of Alanen's Yale thesis. The introduction discusses the use of the computer in attacking number-theoretic problems. He is concerned with the construction and proof of algorithms, and quotes Dijkstra: "Testing shows the presence, not the absence, of bugs." An aliquot sequence ("series") is  $s^0(n) = n$ ,  $s^k(n) = s(s^{k-1}(n))$ , where  $s(n) = \sigma(n) - n$  is the sum of the aliquot parts of  $n$ , the divisors of  $n$  apart from  $n$  itself. He classifies aliquot sequences as purely periodic ("sociable numbers of index  $k$ "), ultimately periodic, and unbounded. He lists the 13 purely periodic sequences then known with period ("index") greater than 2. These are due to Poulet ( $k = 5, 28$ ), Borho, Cohen and David ( $k = 4$ ). Further 4-cycles have since been found by David and by Root. He defines  $s(0) = 0$  and so includes among the ultimately periodic sequences those called terminating by the reviewer and Selfridge [1]. He lists the six known candidates (276, 552, 564, 660, 840, 966) for "main"  $n$  sequences with  $n < 1000$  which may remain unbounded; the reviewer, D. H. Lehmer, Selfridge and Wunderlich [2] have now pursued these to terms 433, 181, 265, 168, 195, and 184, which contain 36, 35, 31, 33, 31, and 32 digits.

He gives properties of  $s(n)$  and points out that it induces a digraph on the nonnegative integers as vertices. There is part of the drawing of the digraph containing the perfect number  $P = 8128$ . He gives the members of the monotonic sequence  $s^k(3P)$ ,  $0 \leq k \leq 48$ . (The sequence  $s^k(9336)$  leads into this  $s^k(3P)$  at  $s^4(9336) = s^3(3P) = 9P$ , and, in [2],  $s^k(3P)$  was thereby continued until

$$s^{96}(3P) = 22\ 14196\ 97766\ 28194\ 23647\ P,$$

where it is still monotonic. The theory of  $s^k(nP)$  with  $n$  odd and  $P$  perfect has been given by te Riele [3]. For large  $P$  and  $n = 27$  the sequence has been continued to  $k = 136$  [3], [4].)

If the invalence of an integer is zero, Alanen calls it "untouchable". He conjectures that no odd number other than 5 is untouchable, and refers to this as a strengthened Goldbach conjecture (or "weakened", since if  $n = p + q + 1$ , where  $p, q$  are distinct odd primes, then  $pq$  is a value of  $s^{-1}(n)$ , but there are in general other than "Goldbach" solutions to the equation  $s(x) = n$ ). He gives the value of the invalence of  $n$  and a list of all solutions of  $s(x) = n$  for  $0 \leq n \leq 100$ , the non-Goldbach solutions for  $101 \leq n \leq 500$ , and a list of the 570 untouchable numbers 2, 5, 52, 88, ... less than 5000.

Two sections develop algorithms for determining all untouchable numbers, and all cycles, below a given bound. Two others give the results of computer calculations (see next review) and a specification of the algorithms used. There are 33 references.

RICHARD K. GUY

University of Calgary  
Calgary, Alberta, Canada, T2N 1N4

1. RICHARD K. GUY & J. L. SELFRIDGE, "Interim report on aliquot series," *Proc. Winnipeg Conf. on Numerical Math.*, October 1971, pp. 557-580.
2. RICHARD K. GUY, D. H. LEHMER, J. L. SELFRIDGE & M. WUNDERLICH, "Second report on aliquot sequences," *Proc. Winnipeg Conf. on Numerical Math.*, October 1973.
3. H. J. J. TE RIELE, "A note on the Catalan-Dickson conjecture," *Math. Comp.*, v. 27, 1973, pp. 189-192.
4. Corrigendum, *ibid.*, p. 1011.

44[9].—JACK ALANEN, *Tables of Aliquot Sequences*, 7 volumes, each of approximately 600 pages of computer output, filed in stiff covers and presented to the reviewer.

This is the output produced in connection with the author's thesis (see previous review). One volume investigates, by various algorithms, (certain subclasses of) partitions of  $n$ , and also carries out other algorithms designed to find all solutions of  $s(x) = n$ , where  $s(x)$  is the sum of the divisors of  $x$ , other than  $x$  itself. A second volume continues the previous work, lists all  $n$  sequences with  $n \leq 48303$  for which  $s^k(n) = 6$  for some  $k$  (the largest  $k$  in this range is 33) and lists the members of the aliquot sequence  $s^k(138)$  for  $0 \leq k \leq 112$  (it was earlier shown by D. H. Lehmer that  $s^{177}(138) = 1$ , the maximum term being  $s^{117}(138) = 179931895322$ ). The other five volumes give all terms of all  $n$  sequences for  $1 \leq n \leq 10000$ ,  $10001 \leq n \leq 20000$ ,  $20001 \leq n \leq 30000$ ,  $30001 \leq n \leq 40000$ ,  $40001 \leq n \leq 48303$  and the rank of the bounding term, where the bounding term is either 1, or a member of a cycle, or the first term of the sequence which exceeds  $10^{10}$ . Of the sequences associated with the first 40,000 integers, 33450 terminate at 1, 5676 exceed Alanen's bound of  $10^{10}$ , 325 become periodic at a perfect number (6, 496 or